

# Study of thermometers for measuring a microcanonical phase transition in nuclear fragmentation

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## Abstract

The aim of this work is to study how the thermodynamic temperature is related to the known thermometers for nuclei especially in view of studying the microcanonical phase transition. We find within the MMMC-model that the "S-shape" of the caloric equation of state  $e^*(T)$  which is the signal of a phase transition in a system with conserved energy, can be seen in the experimentally accessible slope temperatures  $T_{slope}$  for different particle types and also in the isotopic temperatures  $T_{He-Li}$ . The isotopic temperatures  $T_{H-He}$  are weaker correlated to the shape of the thermodynamic temperature and therefore are less favorable to study the signal of a microcanonical phase transition. We also show that the signal is very sensitive to variations in mass of the source.

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In this work we are interested in testing the different experimentally accessible thermometers for nuclei in order to understand which quantity is best related to their thermodynamic temperature. It is our purpose to show that it should be in principle possible to measure the specific signal of a microcanonical phase transition in an accurate experiment.

The concept of phase transitions is usually discussed in connection with macroscopic systems. In such systems phase transitions are recognized by divergences in quantities like specific heat  $c(e^*) = de^*/dT_{thd}$ , where  $e^*$  is the specific excitation energy and  $T_{thd}$  the thermodynamic temperature. If we are interested by similar phenomena in finite systems, the divergences are unsuitable to recognize and to classify phase transitions, since no divergences can occur. In a finite system the conservation laws become significant for the appearance and the shape of a phase transition. If a finite system is in thermal contact with a heat bath of constant temperature  $T_{thd}$ , i.e. is a canonical system, then the signal of a first order phase transition will become smeared showing up as a bump (instead of a divergence) in heat capacity or, equivalent, as a smooth anomaly in the caloric curve  $e^*(T_{thd})$ . For a finite and *isolated*, i.e. microcanonical system the signal changes qualitatively. Here the total energy  $E$  of the system is a strictly conserved quantity. The first order phase transition becomes signaled by an "S-shape" in the caloric equation of state  $T_{thd}(E)$  [1,2], as shown in the following figures in this publication.

Since excited nuclei are an example of finite as well as isolated systems, they are especially suitable to study the signal of a microcanonical phase transition. This signal with an "S-shape" was frequently obtained in calculations, though it is still a subject of controversial discussion. Therefore it is of fundamental interest to test the theoretical findings by an experiment.

Another important point is the fact that the thermodynamic temperature which can be simply measured in macroscopic physics is not accessible directly in a finite system. In a macroscopic system the measurement is simple through the fact that the size of the thermometer which gets into thermal contact with the system of interest is negligible compared to the size of the system. For a finite system this is obviously not satisfied. Especially for a microcanonical system it is not possible to bring a thermometer into thermal contact with a system without violating the strict energy conservation. Thus to obtain information on the thermodynamic temperature one needs to find an experimental observable which is not a temperature, but keeps information on the behavior of  $T_{thd}$ .

Several suggestions for such observables were made. The candidates for "nuclear thermometers" are the slope temperatures from Maxwellian fits of energy distributions and the temperatures deduced from the isotopic ratios. In this work we are testing the quality of these "nuclear thermometers" as concerning their ability to reproduce the shape of the microcanonical caloric equation of state (*CES*)  $T_{thd}(E)$ . Another approach can be found in [14].

We concentrate on the microcanonical phase transition from evaporation to asymmetric fission which was predicted in the fragmentation of hot nuclei within the Berlin statistical fragmentation model MMMC [3]. Similar signals were also predicted by other statistical models for nuclear fragmentation [4,5] and also for atomic clusters [6–8]. An experimental observation in ref. [9] seems also to support the existence of this phase transition.

To study the "nuclear thermometers" we first obtain the signal of a phase transition in the  $T_{thd}(E)$  curve within the MMMC-model. Then we calculate the signals of the two

thermometers in question, the caloric curves of  $T_{slope}(E)$  and  $T_{isotopic}(E)$ .

Let us first briefly discuss the basics of the model. The strictly microcanonical MMMC-model [3] assumes a hot compound nucleus to be formed in a nuclear collision. After fragmentation of this compound system the fragments remain coupled and exchange nuclei as long as they are in close contact. Consequently, the system is assumed to equilibrate statistically shortly after the break-up. The volume which is accessed by the equilibrated fragment configuration is called the freeze-out volume. This means in terms of thermodynamics that the collection of all possible fragment configurations represents the maximum accessible phase space  $\Omega(E)$  for a given freeze-out volume.  $\Omega(E)$  is restricted by the valid conservation laws, which are the conservation of mass, charge, linear and angular momentum and of the total energy of the system, and also by the geometrical constraints. In the simulation the most important geometrical constraint is the size of the freeze-out volume, which is taken to be spherical. The radius  $r_f$  of this volume, which is the only simulation parameter of the MMMC-model, is for the energy region of 1 to 4 MeV per nucleon at about  $r_f = 2.2A^{1/3}\text{fm}$ , which corresponds to approximately 6 times the normal nuclear volume. When the fragments (which can be in excited states) leave this volume they may de-excite as they trace out Coulomb-trajectories.

The output of the MMMC calculation is a collection of freeze-out configurations which are supposed to be representative for the entire phase space  $\Omega(E)$ . The thermodynamic temperature  $T_{thd}$  of these configurations is calculated by

$$\frac{1}{T_{thd}} = \frac{\partial S}{\partial E} = \frac{\partial s}{\partial E^*} , \quad (1)$$

where  $E$  is the total energy of the system,  $S = k_B \ln \Omega(E)$  is the entropy,  $k_B$  the Boltzmann constant and  $s = S/A$  and  $E^* = E/A$  with  $A$  the mass of the decaying nucleus. The caloric curve  $T_{thd}(E^*)$  is plotted in all figures of this paper as a solid line with circles.

We test the "nuclear thermometers" in two steps. First we plot the caloric curves  $T_{slope}(E^*)$  and  $T_{isotopic}(E^*)$  obtained from the MMMC-events after performing the Coulomb trajectories. Next we subject the calculated events to the software filter of the INDRA setup [10] and plot the filtered caloric curves.

For the INDRA-filter we assume the source velocity as 8.1 cm/ns, which is a typical quasi-projectile velocity measured with INDRA in mid-peripheral Xe + Sn collisions at 50 A.MeV bombarding energy [11]. After the filtering we select the complete events for which the detected total charge and the total momentum is greater than 80% of the initial charge and of the initial momentum of the source, respectively. These events are used for constructing the filtered caloric curves.

We start with the slope temperatures  $T_{slope}$  for protons, deuterons, tritons,  $^3\text{He}$  and alpha particles. The calculated kinetic energy spectra were fitted with the surface-evaporating Maxwellian source formula [12] for every particle type:

$$\frac{d\sigma}{dE_{kin}} \propto \frac{(E_{kin} - B)}{T_{slope}^2} e^{(-\frac{E_{kin}-B}{T_{slope}})} , \quad (2)$$

where  $E_{kin}$  is the center of mass kinetic energy of the particles,  $B$  the Coulomb barrier and  $T_{slope}$ , which is the slope of the raw spectra, which is the desired slope temperature. We

calculate the slope temperature for protons, deuterons, tritons  ${}^3\text{He}$ , alpha and also light IMFs.

Figures 1, 2, 3, 4 and 5 show the comparison of  $T_{thd}(E^*)$  and  $T_{slope}(E^*)$  for unfiltered (left plot) and INDRA-filtered (right plot) events for  $p$ ,  $d$ ,  $t$ ,  ${}^3\text{He}$  and  $\alpha$ . Our most important finding in all these curves is that the slope temperatures resemble the general shape of  $T_{thd}(E^*)$  before and after the filtering.

The caloric curves for Li, Be and B (not shown here) which we have calculated only without filtering, repeat also the general shape of the phase transition despite of big error bars in  $T_{slope}$ .

The unfiltered  $T_{slope}(E^*)$  for  $p$ ,  $d$ ,  $t$  systematically achieve values lower than the thermodynamic temperatures, while the values for  ${}^3\text{He}$  and  $\alpha$  are close to those of  $T_{thd}$ . We think that the last finding is just accidental. The unfiltered  $T_{slope}$  are close for all particles with  $Z = 1$  and for those with  $Z = 2$ , and the shift between them of  $\approx 0.3$  MeV is due to the higher Coulomb repulsion. Here one can see in a very simple way that the result of the MMMC calculation cannot be described by a Maxwellian source with a *unique* temperature  $T_{slope}(E^*)$ . Still we see that using the Maxwellian fit just as a recipe we obtain a useful tool to extract a pseudo-temperature which is correlated with the thermodynamic temperature.

Let us now proceed to the isotopic temperatures [13]. The basic assumption for the isotopic temperature formula is a thermal equilibrium between free nucleons and composite fragments within a certain interaction volume  $V$  at a temperature  $T$ . The formalism is of the grandcanonical ensemble and ignores the possible effects of mass and energy conservation.

The  $T_{He-Li}(E^*)$  isotopic temperature is given by

$$T_{He-Li} = 16/\ln(2.18 \frac{Y_{6Li}/Y_{7Li}}{Y_{3He}/Y_{4He}}) \quad , \quad (3)$$

and the  $T_{H-He}(E^*)$  temperature by

$$T_{H-He} = 14.3/\ln(1.6 \frac{Y_{2H}/Y_{3H}}{Y_{3He}/Y_{4He}}) \quad , \quad (4)$$

where  $Y(E^*)$  is the particle yield.

Fig. 6 shows a comparison of unfiltered and filtered  $T_{He-Li}(E^*)$  with the thermodynamic temperature  $T_{thd}(E^*)$ . Again we see that the signal of the phase transition survived the procedure.

Performing the same for the H-He isotopic ratios, figure 7, we find that already the unfiltered  $T_{H-He}(E^*)$  isotopic temperature is less sensitive to the "S-shape" in the  $T_{thd}(E^*)$  curve, but it also shows some structure at the phase transition.

Finally we would like to address the question, how sensitive is the signal of the microcanonical phase transition to the mass and charge of the compound nucleus. Figure 8 shows the  $T_{thd}(E^*)$  curve for several masses and charges of the source. In the left panel we show that decreasing the charge of the source from  $Z=54$  for  ${}^{122}\text{Xe}$  (dots) to  $Z=50$  for  ${}^{122}\text{Sn}$  (empty triangles) does not influence much the phase transition. On the opposite, increasing the mass by 10 nuclei for  ${}^{132}\text{Xe}$  (full triangles) shifts the transition signal to higher excitation energies by approximately 0.5 MeV per nucleon.

In the right panel we show two additional curves for  ${}^{80}\text{Se}$  and  ${}^{250}\text{Cf}$ , thus strongly reducing and strongly decreasing the mass. Here the evaporation-fission phase transitions for  ${}^{80}\text{Se}$

appears as a very weak signal at  $E^* \approx 1.5$  A.MeV and  $T_{thd} \approx 3.8$  MeV. The stronger signal at  $T_{thd} \approx 4.6$  MeV is due to a different fission-multifragmentation phase transition.

The fragmentation behavior of  $^{250}\text{Cf}$  shows no phase transition at all, even for higher or lower  $E^*$ . This is connected to the intrinsic instability against fission, so that even at very low excitation energies the nucleus fissions symmetrically instead of entering first the evaporation mode and later the asymmetric fission mode like the lighter nuclei do. This shows very plastically that there is no unique liquid-gas phase transition in nuclear fragmentation, but many different transitions depending on mass, and may be on other characteristics of nuclei.

The dependence of the phase transition on the mass is natural, since increasing the mass we automatically increase the phase space  $\Omega(E^*)$ . Thus if changing  $A$  with excitation energy can also produce unusual shapes of the  $T_{thd}(E^*)$  curve<sup>1</sup> without undergoing any phase transition. This discussion serves mainly to make the point that to measure the microcanonical phase transition it is essential to keep especially the mass of the source at constant over the whole energy range.

Further, it is important to obtain energy binning less than 0.2 MeV per nucleon to detect the discussed transition. The investigation in ref. [16] misses the phase transition by performing energy steps  $\Delta E^* \approx 1$  A.MeV. The current INDRA setup can realize  $\Delta E^* \approx 0.4$  A.MeV for the energy region of 1 to 4 A.MeV, which is at the limit of desired accuracy. If an additional experimental setup could measure the mass of the projectile fragments one could obtain much better  $E^*$  reconstructions and  $\Delta E^*$  below 0.2 A.MeV.

Summarizing the above we studied how the shape of the caloric curve  $T_{thd}(E^*)$  at a microcanonical phase transition is correlated to the shape of different nuclear thermometers  $T_{slope}(E^*)$  and  $T_{isotopic}(E^*)$ . The signal of a microcanonical phase transition is an "S-shape" in the thermodynamic temperature. The slope temperatures for proton, deuteron, triton,  $^3\text{He}$  and alpha and the isotopic temperature  $T_{He-Li}(E^*)$  reproduce this "S-shape" at correct excitation energies for INDRA-filtered and unfiltered events. The absolute values of this curves vary and do not coincide with the thermodynamic temperature. This means that the decrease of temperature with rising excitation energy at a first order microcanonical phase transition can be measured experimentally. This is of fundamental importance for the systematic study of phase transitions in finite systems.

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<sup>1</sup>We suppose together with [15,16] and in opposite to [17] that the curve shown in ref. [18] is just the effect of changing the mass of the source.

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# FIGURES

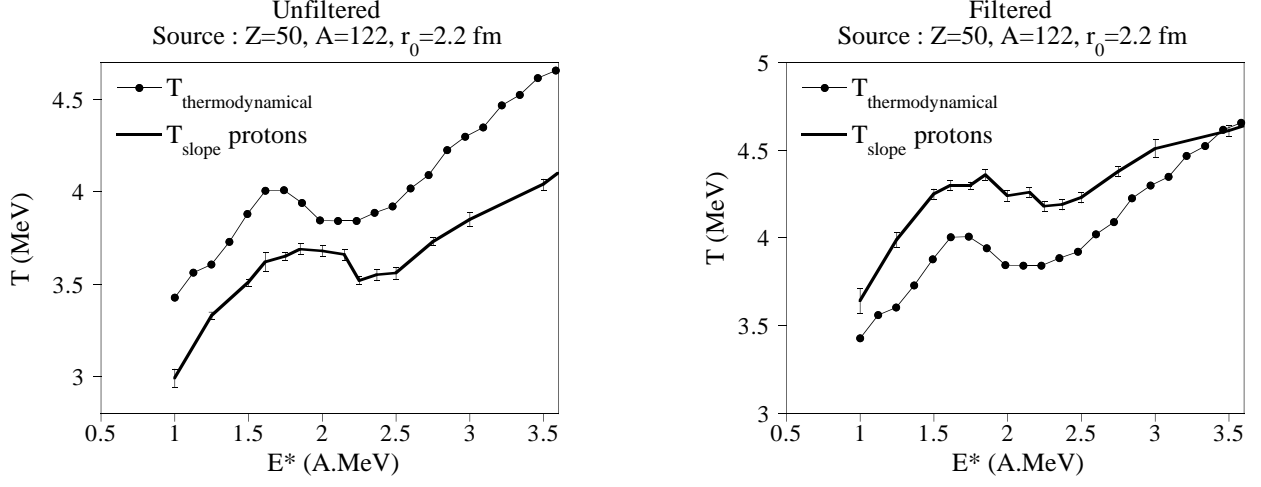


FIG. 1. Caloric curves  $T_{thd}(E^*)$  and  $T_{slope}(E^*)$  for protons. Left panel: MMC calculation,  $T_{slope}(E^*)$  unfiltered. Right panel: MMC calculation,  $T_{slope}(E^*)$  filtered according to the INDRA setup.

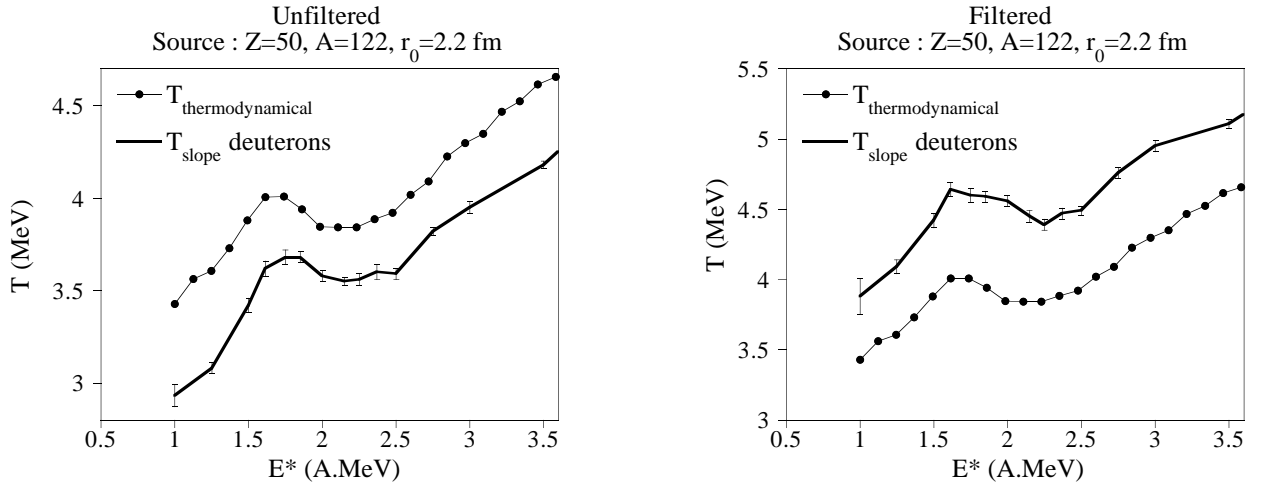


FIG. 2. Like figure 1, but for deuterons.

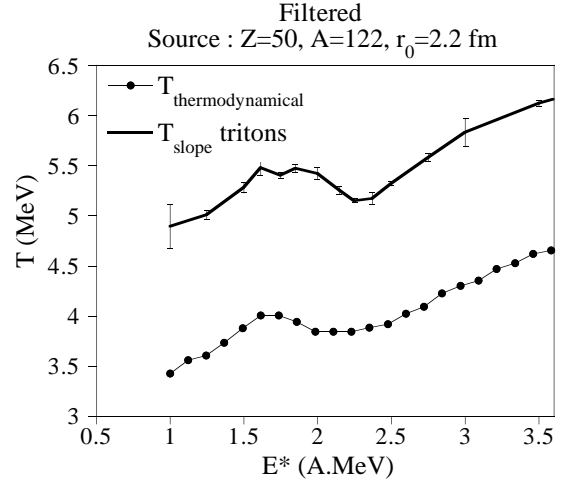
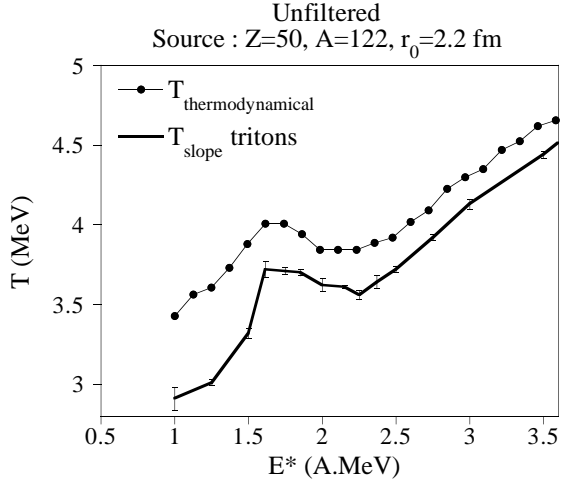


FIG. 3. Like figure 1, but for tritons.

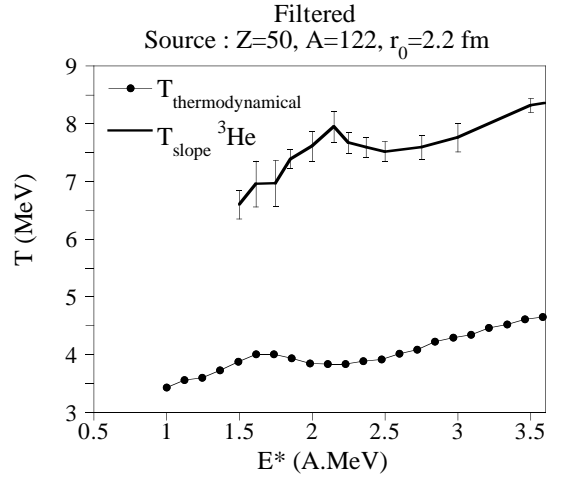
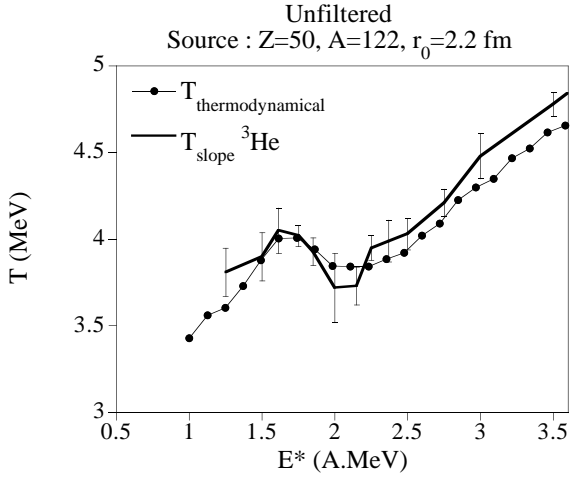


FIG. 4. Like figure 1, but for  $^3\text{He}$



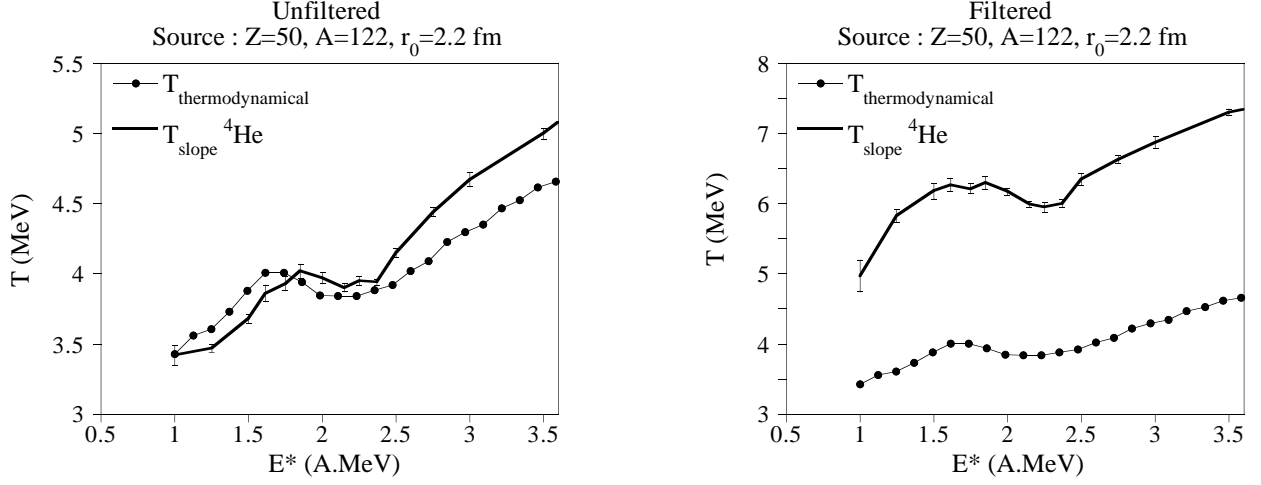


FIG. 5. Like figure 1, but for alpha.

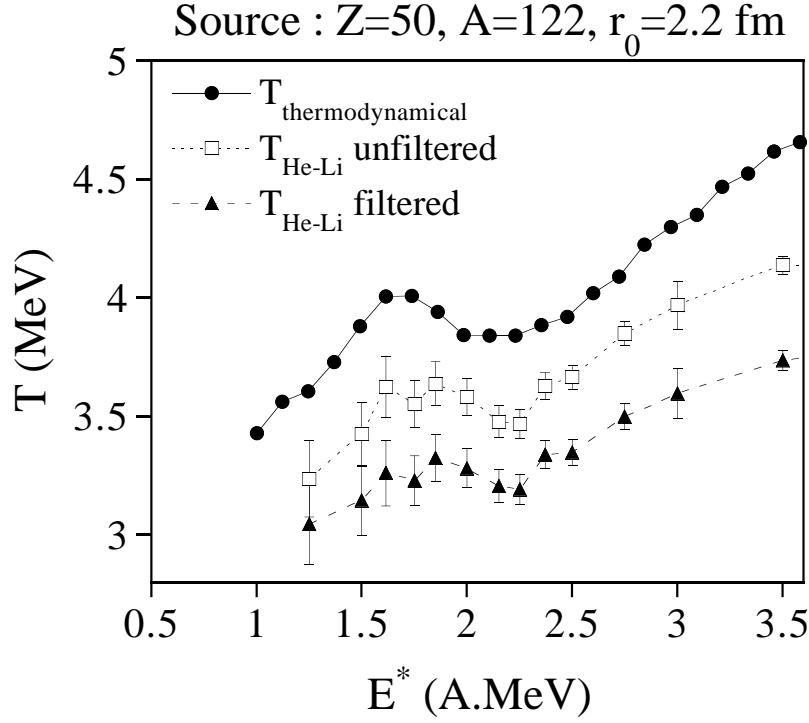


FIG. 6. Caloric curves  $T_{\text{thd}}(E^*)$  and  $T_{\text{He-Li}}(E^*)$  from the MMC calculation.  $T_{\text{He-Li}}$  is shown unfiltered and filtered according to the INDRA setup.

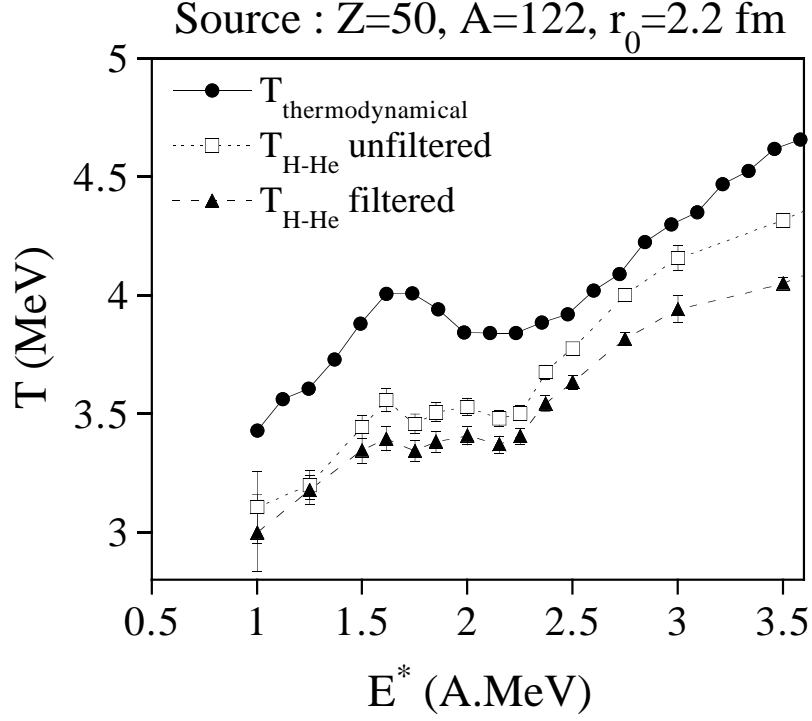


FIG. 7. Like figure 6, but for H-He isotopic temperature  $T_{H-He}(E^*)$ .

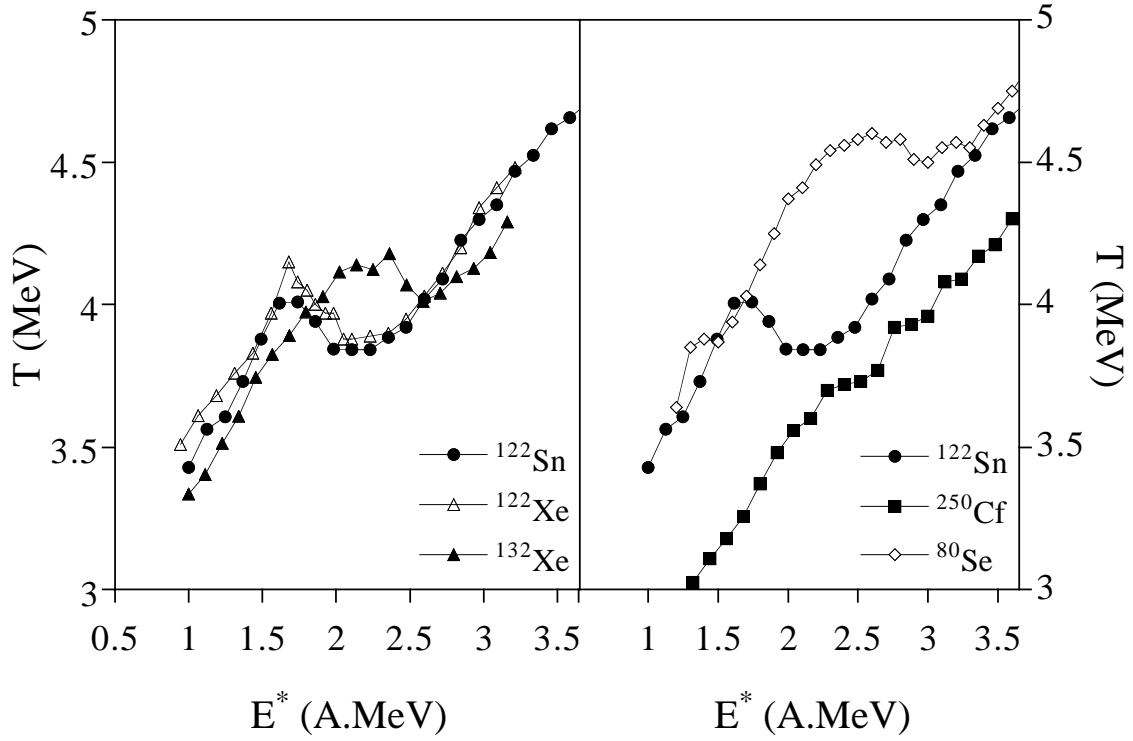


FIG. 8. Signals of phase transitions in the thermodynamic temperature  $T_{thd}(E^*)$  from MMMC calculations for different masses and charges of compound nuclei.